

NEW METHODS IN THE THEORY OF GAUGE FIELDS

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One of the fundamental problems of the theoretical physics is the search of the axioms, which ought to be the basis for the one-valued construction of Lagrangians of the relativistic fields. The creation of the gauge fields theory was the great success in the solution of this problem. The gauge formalism allowed to derive the total Lagrangians of the interacting fields from the postulated Lagrangians of the noninteracting (free) fields. We offer to do quite the reverse in consequence of what it is necessary to seek from the out set the construction principles of the total Lagrangians. By the theory construction we shall differ the wave-functions being the solutions of the differential equations (“theoretical” functions) from the wave-functions which is constructed on the base of the experimental data possibly received by a scattering of particles (“empiric” functions). The “empiric” functions are necessary only for the definition (it is possibly only approximately) of the transition operators which will affect at the “theoretical” functions. This operators will be approximated the differential operators so, that the generalized variance of the differentiable “theoretical” fields will be the minimal one.

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1. Introduction

As is known the experimentators spend many efforts to relieve of the useless information (of the noise). It can assume in this aspect what the human brain is achieving the more, in consequence of what a major proportion of the Universe is showed for us as an empty space with the Euclidean properties, if any. We observe the presence of the matter only here and there so what it is necessary to ask: is the observable (of course also and by instruments) matter the main form or it play the off-beat role of the Brownian particles? This question became particularly a high-priority task since it was established the discrepancy of the galaxies kinetic energies and the galaxies potential energies with the virial theorem [1]. Among the different hypothesis explaining this discrepancy we derive only those in which it was suggested to consider the neutrinos (under which we shall imply antineutrinos, too) as the fundamental form of the matter (it is necessary to note in consequence of this the Wheeler's book [2]). In particular Pontekorvo and Smorodinskij [3] explained the charge asymmetry of the baryon matter that it is only a slight fluctuation at the giant neutrinos background of the Universe. Having the neutrinos Universe and taking account of the Fermi-Dirac statistics we can remind about the Sakharov hypothesis [4] in which the vacuum elasticity is connected with the gravitational interactions. As a result there is a chance to replace the vacuum elasticity by the neutrinos matter elasticity.

So it was necessary to show that the gravitation interaction is not a fundamental one, but the one is induced by others interactions as possible hypothetical ones. The more so, that the gravitational constant $G_N \approx 6.7 \cdot 10^{-39} \text{ GeV}^{-2}$ (it is used the system of units $\hbar = c = 1$, where $2\pi\hbar$ is the Planck's constant and c is the light speed) is a suspiciously small value and a dimensional one furthermore (as is known the latter prevent to the construction of the renormalizable quantum theory). Before building the theory of the induced gravitation on the base of the hypothetical interactions and the hypothetical particles it was necessary to verify the possibility of the utilization of the known particles and the known interactions for this purpose. Naturally that the neutrinos are the most suitable particles for this taking into account their penetrating ability, which allow them to interact with all the substance of the macroscopic body — not with the surface layer only. As is known [5], already in 30th Gamow and Teller offered to use the neutrinos for the explanation of the gravitation, but their mechanism provided the direct exchange of the pairs consisting of a neutrino and an antineutrino.

Bashkin's works appearing in 80th on a propagation of the spin waves in the polarized gases [6] allowed to make the supposition [7], that the analogous collective oscillations are possible under certain conditions as well as in the neutrinos medium. If we shall consider that the effective temperature of the Universe neutrinos is the fairly low one then it is fulfilled one of conditions ($\lambda \gg r_w$, where λ is the de Broglie's wave-length of a neutrino and r_w is the weak interaction radius of an one [6]) of the propagation of the spin wave in the polarized gases. As a result the quantum effects become the determining ones in such medium

and the interference of the neutrinos fields (being the consequence of the known identity of elementary particles) must induce the quantum beats, which will be interpreted as zero oscillations of a vacuum. In consequence of this the mathematical apparatus [8] applied by the description of the Casimir's effect [9] can be used.

We shall be interesting in quantum beats arising by the interference of the falling polarized flow of the Universe neutrinos on the macroscopic body with the scattered one at this body. Let's suppose for this the neutrinos have the zero rest mass (the other version [10] will not be considered), so that the direction of their spin is connected hardly with the direction of their 3-velocity. In consequence of this only those neutrinos can be considered as ones forming the polarized flow, which propagate along straight line connecting specifically two particles of different macroscopic bodies. It explain the anisotropy of the zero quantum oscillations, which is necessary to obtain the right dependence ($1/R$) of the energy of the two-particles interaction on the distance R between particles in the Casimir's effect.

Let's consider two macroscopic bodies with masses m_1 and m_2 and with the fairly long distance R one from another [11]. We shall regard, that the bodies contain $2m_1r_w$ and $2m_2r_w$ particles correspondingly (where the normalizing factor r_w is connected with cross-section σ_ν of the neutrino upon the particle), implying thereby the statistics averaging of the properties of the elementary particles constituting the bodies. If the particles of the macroscopic bodies had interacted with all neutrinos incidenting on them then these particles might have been considered as the opaque boardaries, which induce Casimir's effect on the straight line. By this the energy of the interaction of the particles would have been equal to [8]

$$\begin{aligned}\varepsilon_{AB} &= \frac{1}{2} \sum_{n=1}^{\infty} \frac{\pi n}{R_{AB}} - \frac{1}{2} \int_0^{\infty} \frac{\pi x}{R_{AB}} dx = \\ &= \frac{i}{2} \int_0^{\infty} \frac{\pi(it)/R_{AB} - \pi(-it)/R_{AB}}{\exp(2\pi t) - 1} dt = -\frac{\pi}{24R_{AB}}\end{aligned}\quad (1.1)$$

(A is a number of a particle of the first macroscopic body and B is a number of a particle of the second body). On account of the weakness of the interaction of neutrinos with particles we are confined to a first approximation, so that the energy E of the interaction of two macroscopic bodies is equal to

$$E \approx \sum_{A=1}^{2m_1r_w} \sum_{B=1}^{2m_2r_w} \varepsilon_{AB}. \quad (1.2)$$

Neglecting the dimensions of the bodies in comparison with interval R between them ($R_{AB} \approx R$), we shall have finally

$$E \approx -2m_1r_w 2m_2r_w \pi/(24R) = -G_N m_1 m_2 / R \quad (1.3)$$

where

$$G_N = \pi r_w^2 / 6 = \sigma_\nu / 6. \quad (1.4)$$

If now we shall be based on the results of the experiments fixing the equality of the gravitation mass and the inert one then it can consider that the spectrum of the particle masses is defined by their interaction with the neutrinos background of Universe. This statement is confirmed what the rest mass of the photon is equal to zero in contradistinction to the masses of the vector bosons W^+ , W^- , Z^0 which interact with the neutrinos immediately [12].

But the main idea is it now for us what the normal matter (not neutrinos) acts as the Brownian particles by the help of which it can make the attempt to estimate the statistic characteristics of the Universe neutrinos background. So from the formula (1.4) it can receive the estimate of the averaged neutrinos energy: $\omega_\nu \sim 10^{-13} \text{ GeV}$ [11], what is sufficiently close to the temperature ($\approx 2.3 \cdot 10^{-13} \text{ GeV}$) of the Universe relict photons.

2. The Lie local loop

We shall rely on the approach suggested by Schrödinger [13] which introduced the set of the unorthogonal to each other wave-functions describing the unspreading wave packet for a quantum oscillator. Later Glauber [14] showed a scope for a description of coherent phenomena in the optics by the Schrödinger introduced states and it was he who called them as coherent. This approach received the further development in Perelomov's work's who proposed the definition of the generalized coherent states specifically as the states arising by the action of the representation operator of a some transformation group on any fixed vector in the space of this representation [15].

It is what allow to give the physical interpretation to the gauge transformations by our opinion as the transformations inducing the generalized coherent states, which are characterized by the continuous parameters [16]. Admittedly if the parameters space are not the compact one (we shall consider the space-time manifold always as its subspace) then by the rather large changes of parameters it is necessary take into account the speed finity of the information propagation in consequence of what the coherence of the states are able to lose (what lead to the absence of the quantum phenomena on the macroscopic level). It makes us change the Perelomov definition considering it taken place for the arbitrary group only in the neighbourhood of identity, what gives rise to generalize the given definition not only for the Lie local groups but and for the Lie local loops.

Let us to consider the wave packet the equivalence relation for the functions $\Psi(\omega)$ of which we shall give by the infinitesimal transformations

$$\Psi \longrightarrow \Psi + \delta\Psi = \Psi + \delta T(\Psi), \quad (2.1)$$

where δT is the particular case of the transition operator (at the beginning the symmetry type is not being specified) and ω is a set of parameters characterizing the generalized coherent state.

As the experiments on the scattering of particles in which the laws of the conservation are being prescribed are the sole source of the information about the structure of the space-time

manifold on the microscopic level taking into account the Noether's theorem we introduce the finite-dimensional manifold M_r of parameters ω^a ($a, b, c, d, e = 1, 2, \dots, r$), connecting its dimensionality r with the numbers of the conserved dynamical invariants. Further we shall consider the space M_r as the manifold, the parameters ω^a as the coordinates of the point $\omega \in M_r$ and we shall give the fields $\Psi(\omega)$ in a certain domain Ω_r of the given manifold. We choose the arbitrary point e in the domain Ω_r and we shall consider that this point e is the centre of the coordinate system. Let the domain Ω_r contain the subdomain Ω_n with the point e by this the domain Ω_n belong to a certain differentiable manifold M_n (although it is possible in is convenient to define the manifold M_n separately from the manifold M_r). Let moreover the set of the smooth curves belonging to the manifold M_n have the common point e . Define also the set of the vector fields $\xi(x)$ being tangents to this curves and we shall consider that a point $x \in \Omega_n$ and on the domain Ω_n the own coordinate system is defined. It is convenient to give the differentiable manifold M_n by the differentiable functions $\omega^a = \omega^a(x)$ in this case.

Let $\delta\Omega_r$ is the sufficiently small neighbourhood of the point e , thereby and the sufficiently small neighbourhood $\delta\Omega_n$ of the point x is being given ($x \equiv e \in \delta\Omega_n \subset \delta\Omega_r$). The coordinates of the point x note as x^i ($i, j, k, l, p, q = 1, 2, \dots, n$). Further we shall consider the fields $\Psi(x)$ as the cross section of the vector fiber bundle E_{n+N} . Using the vector fields $\delta\xi(x)$ the coordinates of the neighbouring point $x' = x + \delta x \in \delta\Omega_n$ are written down as

$$x'^i = x^i + \delta x^i \cong x^i + \delta\omega^a \xi_a^i(x). \quad (2.2)$$

Comparing the values of the fields $\Psi'(x)$ and $\Psi(x')$, where

$$\Psi'(x') = \Psi + \delta\Psi = \Psi + \delta T(\Psi) \cong \Psi + \delta\omega^a T_a(\Psi), \quad (2.3)$$

$$\Psi(x') = \Psi(x + \delta x) \cong \Psi + \delta\omega^a \xi_a^i \partial_i \Psi \quad (2.4)$$

(∂_i are the partial derivatives), we see that they are differing by the observables

$$\delta_o \Psi(x) \cong \delta\omega^a X_a(\Psi) = \delta\omega^a [T_a(\Psi) - \xi_a^i \partial_i \Psi], \quad (2.5)$$

which can interpret as the deviations the field $\Psi(x)$, received with the help of the transformations (2.3). Further we shall consider the domain $\delta\Omega_r \subset M_r$ as the domain of the Lie local loop G_r (specifically which can have and the structure of the Lie local group if we provide it with the property of the associativity) with the unit e induced by the set $\{T\}$, by this we shall consider the expression of the form (2.3) as the infinitesimal law of the transformations of the Lie local loop of the fields $\Psi(x)$. Precisely the structure of the Lie local loop will characterize the degree of the coherence considered by us the quantum system. By this the maximal degree is being reached for the Lie simple group and the minimal degree is being reached for the Abelian one. In the last case we shall have the not coherent mixture of the wave-functions, it's unlikely which can describe the unspreading wave packet that is being

confirmed by the absence of the fundamental scalar particles, if hipothetical particles are not being taken into account (in experiments only the mesons, composed from the quarks, are being observed and which are not being considered the fundamental one). Note that the “soft” structure of the Lie local loop by contrast to the Lie group allow to use it by the description of the symmetry both the phase transition (there is the time dependence) and the compact objects (there is the space dependence) especially.

As it's unlikely it can be to ignore an interaction between particles we must be able to select those interactions which interest us. Precisely therefore it makes sense to select the set of the operators which will play the role of the connection in further. Of course we take into account the dependence of the reference systems on the physical properties of the instruments (including the primary standards) and moreover what the part of the transition are not the observable ones. Let

$$L_a(\Psi) = T_a(\Psi) + \xi_a^i \Gamma_i \Psi. \quad (2.6)$$

In consequence of this the formula (2.6) is rewritten so

$$\delta_o \Psi \cong \delta \omega^a X_a(\Psi) = \delta \omega^a [L_a(\Psi) - \xi_a^i \nabla_i \Psi], \quad (2.7)$$

where ∇_i are the covariant derivatives with respect the connection $\Gamma_i(x)$. Note, if $L_a(\Psi) = L_a \Psi$, then the following relations [17]

$$\xi_a^i \nabla_i \xi_b^k - \xi_b^i \nabla_i \xi_a^k - 2 S_{ij}^k \xi_a^i \xi_b^j = -C_{ab}^c \xi_c^k, \quad (2.8)$$

$$L_a L_b - L_b L_a - \xi_a^i \nabla_i L_b + \xi_b^i \nabla_i L_a + R_{ij}^k \xi_a^i \xi_b^j = C_{ab}^c L_c \quad (2.9)$$

must take place, where $S_{ij}^k(x)$ are the components of the torsion of the space-time M_n

$$S_{ij}^k = (\Gamma_{ij}^k - \Gamma_{ji}^k)/2 \quad (2.10)$$

and $R_{ij}(x)$ are the components of the curvature of the connection $\Gamma_i(x)$

$$R_{ij} = \partial_i \Gamma_j - \partial_j \Gamma_i + \Gamma_i \Gamma_j - \Gamma_j \Gamma_i. \quad (2.11)$$

Here and further $\Gamma_{ij}^k(x)$ are the components of the internal connection of the space-time M_n . By this the components $C_{ab}^c(x)$ of the structural tensor of the Lie local loop G_r must satisfy to the identities

$$C_{ab}^c + C_{ba}^c = 0, \quad (2.12)$$

$$C_{[ab}^d C_{c]d}^e - \xi_{[a}^i \nabla_{|i|} C_{bc]}^e + R_{ij[a}^e \xi_b^i \xi_c^j] = 0, \quad (2.13)$$

where $R_{ija}^e(x)$ are the components of the curvature of the connection $\Gamma_{ia}^b(x)$

$$R_{ijb}^a = \partial_i \Gamma_{jb}^a - \partial_j \Gamma_{ib}^a + \Gamma_{ic}^a \Gamma_{jb}^c - \Gamma_{jc}^a \Gamma_{ib}^c. \quad (2.14)$$

We construct the differentiable manifold M_n , not interpreting it by physically. Of course we would like to consider the manifold M_n as the space-time M_4 . At the same time it is impossible to take into account the possibility of the phase transition of a system as a result of which it can expect the appearance of the coherent states. In consequence of this it is convenient do not fix the dimensionality of the manifold M_n . It can consider that the macroscopic system reach the precisely such state by the collapse. As a result we have the classical analog of the coherent state of the quantum system. Besides there is the enough developed apparatus — the dimensional regularization using the spaces with the changing dimensionality and representing it only on the microscopic level.

3. The gauge fields

We shall demand the minimality of the variations (2.5) (or (2.7)), if only on the “average”, in order to can be hope for the set of the fields $\Psi(x)$ is capable to describe the wave packet. Consider for this the following integral

$$\mathcal{A} = \int_{\Omega_n} \mathcal{L} d_n V = \int_{\Omega_n} \kappa \overline{X^b}(\Psi) \rho_b^a X_a(\Psi) d_n V, \quad (3.1)$$

being the analogue of the fields $\Psi(x)$ variance in the domain Ω_n at issue, which we shall call the action, and \mathcal{L} we shall call the Lagrangian. Here and further $\rho_a^b(x)$ are the components of the density matrix $\rho(x)$ (note that Latin indexis are the only (possible) visible part of the indexis of the density matrix, $\text{tr } \rho = 1$, $\rho^+ = \rho$, the top index + is the symbol of the Hermitian conjugation), and the bar means the Dirac conjugation which is the superposition of the Hermitian conjugation and the space inversion. Solutions $\Psi(x)$ (and even one solution) of equations, which are being produced by the requirement of the minimality of the integral (3.1) can be used for the constraction of the all set of the functions $\{\Psi(x)\}$ (generated by the transition operator), describing the wave packet. It is naturally to demand the invariance of the integral (3.1) relatively the transformations (2.2) and (2.3), in consequence of what it is necessary to introduce the additional fields $B(x)$ with the transformation law in point $x \in \delta\Omega_n$ in the form

$$\delta_o B = \delta\omega^a Y_a(B) + \nabla_i \delta\omega^a Z_a^i(B), \quad (3.2)$$

and which we shall name the gauge ones. Make it in the standard manner defining them by the density matrix $\rho(x)$ as

$$B_\gamma^b \overline{B_a^\gamma} = \rho_a^b(B_\gamma^c \overline{B_c^\gamma}). \quad (3.3)$$

by this the factorization of the gauge fields $B(x)$ on equivalence classes is allowed for the writing of the indexes of their components $B_\alpha^a(x)$. Note, that $B_\alpha^a(x)$ can be both Utiyama gauge fields [18] and Kibble gauge fields [19]. Following for Utiyama [18] we shall not concretize significances which are adopted by the Greek indexes.

Further we shall assume that the density matrix $\rho(x)$ defines the dimensionality of manifold M_n , using even if for this the corresponding generalized (singular) functions in consequence of what the rank of the density matrix $\rho(x)$ must be equal to n , and the formula (3.1) can be rewritten in the form

$$\mathcal{A} = \int_{\Omega_r} \mathcal{L} d_r V = \int_{\Omega_r} \kappa \overline{X^b}(\Psi) \rho_b^a X_a(\Psi) d_r V, \quad (3.4)$$

We should connect the rank n of the density matrix $\rho(x)$ with the nonzero vacuum average of the gauge fields B_α^a .

In consequence of (3.1) and (3.3) it can consider that the Lagrangian \mathcal{L} depend on the gauge fields B by

$$D_\beta \Psi = -B_\beta^a X_a(\Psi), \quad (3.5)$$

We see by this formula, that the particles charges define the form of the generators $X_a(\Psi)$ and the structural tensor components $C_{ab}^c(x)$ of the Lie local loop G_r , and hence it follows the dependence of the symmetries on the particles charges as and in the Utiyama formalism [18]. If now we shall have “spread” the gauge fields but retaining the terms L_i responsible for the vacuum oscillation, then the Lagrangian \mathcal{L} will have been rewritten in the form

$$\mathcal{L} = k(\partial_i \overline{\Psi} - \overline{L_i \Psi}) \eta^{ij} (\partial_j \Psi - L_j \Psi). \quad (3.6)$$

In particular by $n = 4$ and considering the CPT-degeneracy

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \psi_L = \frac{1}{2}(I - \gamma_5)\psi, \quad \psi_R = \frac{1}{2}(I + \gamma_5)\psi, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad (3.7)$$

$$L_A = -L_A^+ = i\frac{\omega_o}{2}\Sigma_A \otimes \begin{pmatrix} I - \gamma_4 & 0 \\ 0 & I + \gamma_4 \end{pmatrix}, \quad \Sigma_A = \begin{pmatrix} \sigma_A & 0 \\ 0 & \sigma_A \end{pmatrix}, \quad (3.8)$$

$$L_4 = -L_4^+ = i\frac{\omega_o}{2}\gamma_4 \otimes \begin{pmatrix} I - 3\gamma_4 & 0 \\ 0 & I + 3\gamma_4 \end{pmatrix} \quad (3.9)$$

($i^2 = -1$; I is the unit matrix; $\gamma_5 = -i\gamma_1\gamma_2\gamma_3\gamma_4$; $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are the Dirac matrices; σ_A are the Pauli matrices; $A, B = 1, 2, 3$; η^{ij} are the contravariant components of the metric tensor of the Minkowski space; ω_o is a constant) it can obtain the Lagrangian \mathcal{L} of the neutrinos fields in the standard form ($\omega_o = 1/k$)

$$\mathcal{L} = -ik\frac{\omega_o}{2}[\eta^{AB}(\partial_A \overline{\psi} \gamma_B \psi - \overline{\psi} \gamma_A \partial_B \psi) - \partial_4 \overline{\psi} \gamma_4 \psi + \overline{\psi} \gamma_4 \partial_4 \psi] \quad (3.10)$$

If now we turn on the mixing of the fields with different polarization, it is possible substituting L_4 (the formula (3.9)) in the following form

$$L_4 = -L_4^+ = i \frac{\omega_o}{2} \gamma_4 \otimes \begin{pmatrix} (I - 3\gamma_4) & 2I\omega_1/\omega_o \\ 2I\omega_1/\omega_o & (I + 3\gamma_4) \end{pmatrix}, \quad (3.11)$$

then the fields $\Psi(x)$ will describe the particles with the nonzero rest masses. Of course this mixing is the detector of the vacuum frequency change, which is induced by the presence of the added fields (in particular, by the presence of the electromagnetic field).

Now one may proceed to a construction of the covariant gauge formalism considering that the manifold M_n is the Riemannian space-time. For this it is necessary to find a law of a transformation of the fields $B(x)$. Let the fields $D_\alpha \Psi$ change analogously to the fields $\Psi(x)$ in a point $x \in M_n$, then is

$$\delta_0 D_\alpha \Psi = \delta \omega^b (L_b D_\alpha \Psi - L_{b\alpha}^\beta D_\beta \Psi - \xi_b^i \nabla_i D_\alpha \Psi). \quad (3.12)$$

As a result $\delta_0 B_\alpha^a$ are written down in the form:

$$\delta_0 B_\alpha^d = \delta \omega^b (C_{cb}^d B_\alpha^c - L_{b\alpha}^\beta B_\beta^d - \xi_b^i \nabla_i B_\alpha^d) + \Phi_\alpha^i \nabla_i \delta \omega^d, \quad (3.13)$$

where

$$\Phi_\beta^i = B_\beta^a \xi_a^i, \quad (3.14)$$

Since the action

$$\mathcal{A}_t = \int_{\Omega_n} \mathcal{L}_t \eta \, dx^1 dx^2 \dots dx^n \quad (3.15)$$

(Ω_n is a region of the space-time M_n and $\eta(x)$ is the base density of the same) must be invariant against infinitesimal transformations of the Lie local loop G_r , then the total Lagrangian \mathcal{L}_t depending on fields $\Psi(x)$, $B(x)$ and also their derivatives of the first order is unable to be selected arbitrarily. The following Lagrangian $\mathcal{L}_t(\Psi; D_\alpha \Psi; F_{\alpha\beta}^c)$ satisfy to this demand, where the components $F_{\alpha\beta}^c(x)$ of the intensities of the gauge fields $B(x)$ have the form:

$$F_{\alpha\beta}^c = [\delta_b^c - \xi_b^i \Phi_i^\gamma (B_\gamma^c - \beta_\gamma^c)] [\Phi_\alpha^j \nabla_j B_\beta^b - \Phi_\beta^j \nabla_j B_\alpha^b - B_\alpha^e B_\beta^d C_{ed}^b + (B_\alpha^e L_{e\beta}^\delta - B_\beta^e L_{e\alpha}^\delta) B_\delta^b]. \quad (3.16)$$

Note that the fields $\Phi_i^\alpha(x)$ are defined from the equations: $\Phi_\alpha^i \Phi_j^\alpha = \delta_j^i$ (δ_i^j and δ_a^b are the Kronecker delta symbols). The components $\beta_\alpha^b(x)$ of linear homogeneous geometrical objects are being interpreted as vacuum averages of gauge fields $B(x)$. In consequence of $\beta_\alpha^b(x) \neq 0$ the matrixes L_i in the formula (3.6) proved to be the non-zero ones.

Rewrite the equations

$$\Phi_\alpha^i \left(\frac{\mathcal{L}_t}{\eta} \frac{\partial \eta}{\partial B_\alpha^b} + \frac{\partial \mathcal{L}_t}{\partial B_\alpha^b} - \nabla_j \left(\frac{\partial \mathcal{L}_t}{\partial \nabla_j B_\alpha^b} \right) \right) = 0 \quad (3.17)$$

of gauge fields in the quasi-maxwell form:

$$\nabla_j H_a^{ji} = I_a^i, \quad (3.18)$$

where

$$H_a^{ij} = -\Phi_\beta^i \frac{\partial \mathcal{L}_t}{\partial \nabla_j B_\beta^a} = \Phi_\beta^j \frac{\partial \mathcal{L}_t}{\partial \nabla_i B_\beta^a}, \quad (3.19)$$

$$I_a^i = -\mathcal{L}_t \xi_a^i - \frac{\partial \mathcal{L}_t}{\partial \nabla_i \Psi} X_a(\Psi) - \frac{\partial \mathcal{L}_t}{\partial \nabla_i B_\beta^b} Y_{a\beta}^b(B), \quad (3.20)$$

$$Y_{a\gamma}^b(B) = C_{ca}^b B_\gamma^c - L_{b\alpha}^\beta B_\beta^d - \xi_a^i \nabla_i B_\gamma^b. \quad (3.21)$$

Besides let

$$\Phi_\alpha^k \frac{\partial \eta}{\partial B_\alpha^b} + \eta \xi_b^k = 0. \quad (3.22)$$

We pick out from the equations of gauge fields folding them with $B_\alpha^b \Phi_l^\alpha$ those which can will be called the equations of fields $\Phi_\alpha^i(x)$ and which must substitute for Einstein gravitational equations.

Further we shall consider the making of most important equations within the scope of the offered gauge field theory. Note that the base physical workload will impinge on the density matrix, the choice of which will be defined by the symmetries of the physical system states.

4. The Einstein gravitational equations

Let $n = 4$ and the Greek indexes take the values 1, 2, 3, 4. Let moreover $\eta_{\alpha\beta}$ are the covariant components and $\eta^{\alpha\beta}$ are the contravariant components of the metric tensor of the Minkowski space. Introduce the geometrical objects by definitions

$$h_\alpha^i = \beta_\alpha^a \xi_a^i, \quad \eta^{ij} = \eta^{\alpha\beta} h_\alpha^i h_\beta^j, \quad g^{ij} = \eta^{\alpha\beta} \Phi_\alpha^i \Phi_\beta^j \quad (4.1)$$

and define also the geometrical objects $h_\alpha^i(x)$, η_{ij} , g_{ij} as the solutions of the equations

$$h_k^\alpha h_\alpha^i = \delta_k^i, \quad \eta^{ij} \eta_{kj} = \delta_k^i, \quad g^{ij} g_{kj} = \delta_k^i. \quad (4.2)$$

We shall consider that g_{ij} are the covariant components of the metrical tensor of the Riemannian space-time M_4 in consequence of what

$$\nabla_k g_{ij} = 0, \quad \nabla_k g^{ij} = 0. \quad (4.3)$$

As a result the construction of the differentiable manifold M_4 can be connected with the finding of the equations solutions of the gauge fields $\Phi_\beta^i = B_\beta^a \xi_a^i$ received from the demand of the minimality of the total action

$$\mathcal{A}_t = \int_{\Omega_4} \mathcal{L}_t d_4 V = \int_{\Omega_4} [\mathcal{L}(\Psi, D_\alpha \Psi) + \mathcal{L}_1(F_{\alpha\beta}^a)] d_n V, \quad (4.4)$$

where (in particular $k_1 = 1/(16\pi G_N)$)

$$\mathcal{L}_1(F_{\alpha\beta}^a) = \kappa_1 \overline{F_d^{\beta\delta}} \rho_{1\beta\delta b}^d{}^{\alpha\gamma} F_{\alpha\gamma}^b. \quad (4.5)$$

It is naturally that the supposition about fields and particles (the neutrinos) filling the Universe and defining the geometrical structure of the space-time manifold, allow to introduce the connection of the fundamental tensor of this manifold with that kind of the statistical characteristic as the entropy defining it in a standard manner by the reduced density matrix $\rho'(x)$ in the form

$$S = -\text{tr}(\rho' \ln \rho'), \quad (4.6)$$

where the components of the reduced density matrix $\rho'(x)$ are defined as

$$\rho_i^j = \overline{\xi_i^b} \rho_b^a \xi_a^j / (\overline{\xi_k^d} \rho_d^c \xi_c^k). \quad (4.7)$$

As a result the transition from the singular state of the Universe to the modern one with the non-zero vacuum averages β_α^b of the gauge fields B_α^b must be defined by the growth of the entropy S .

Further we shall consider for a simplification of a calculation that the Lie transitiv local loop G_r act effectively in the considered domain of the space-time M_4 , in consequence of this $r = 4$ and let

$$L_{c\alpha}^\beta = 0. \quad (4.8)$$

As a result the formula (3.21) is rewritten as

$$Y_{a\gamma}^b(B) \longmapsto Y_{i\gamma}^k = -\nabla_i \Phi_\gamma^k. \quad (4.9)$$

Write down the Lagrangian \mathcal{L}_1 in the form

$$\mathcal{L}_1 = \kappa_1 \eta^{\alpha\beta} Y_{j\beta}^i(\Phi) Y_{i\alpha}^j(\Phi), \quad (4.10)$$

considering that the fields $\Phi(x)$ satisfy the Lorentz conditions:

$$\nabla_i \Phi_\alpha^i = 0 \quad (4.11)$$

and $\eta^{\alpha\beta}$ are the contravariant components of the metric tensor of the Minkowski space. In this case the Lagrangian \mathcal{L}_1 is distinguished only the constant factor $(-\kappa_1)$ from the scalar curvature $R = g^{ij} R_{ij}$ where $R_{ij} = R_{kij}{}^k$ are the components of the Ricci tensor. It can note that in this case the gauge fields equations are written down as the Einstein equations, namely

$$I_k^j = \kappa_1 (2g^{jl} R_{kl} - \delta_k^j R). \quad (4.12)$$

where the energy-momentum tensor of the everybody fields (excluding the fields Φ_α^i) has the form

$$I_k^j = -\delta_k^j \mathcal{L} + \frac{\partial \mathcal{L}}{\partial \Phi_\alpha^k} \Phi_\alpha^j. \quad (4.13)$$

If we do not wish to use the Lorentz' conditions (4.11), then it can take the following Lagrangian \mathcal{L}_1

$$\mathcal{L}_1 = \frac{1}{2}\kappa_1\eta^{\alpha\beta}\eta_{ijpq}\eta^{klpq}Y_{k\beta}^i(\Phi)Y_{l\alpha}^j(\Phi), \quad (4.14)$$

where η^{klpq} is the base 4-vector of the manifold M_4 and η_{ijpq} is the mutual one to η^{klpq} . As a result

$$\mathcal{L}_1 = \kappa_1\eta^{\alpha\beta}(\nabla_j\Phi_\beta^i\nabla_i\Phi_\alpha^j - \nabla_i\Phi_\beta^i\nabla_j\Phi_\alpha^j), \quad (4.15)$$

It can rewrite this Lagrangian (4.15) also in the form

$$\mathcal{L}_1 = \frac{1}{4}\kappa_1\eta^{\alpha\beta}(F_{\gamma\beta}^\delta F_{\mu\alpha}^\nu\eta^{\gamma\mu}\eta_{\delta\nu} + 2F_{\gamma\beta}^\delta F_{\delta\alpha}^\gamma - 4F_{\gamma\beta}^\gamma F_{\delta\alpha}^\delta) \quad (4.16)$$

where the components $F_{\mu\alpha}^\nu$ of the intensities of the gauge fields $\Phi(x)$ can be got from the intensities (3.16) as

$$F_{\alpha\beta}^\nu = \Phi_k^\nu(\Phi_\alpha^i\nabla_i\Phi_\beta^k - \Phi_\beta^i\nabla_i\Phi_\alpha^k). \quad (4.17)$$

In the more general case, when $r \geq 4$, it ought to use the following total Lagrangian \mathcal{L}_t (in particular $k_1 = 1/(16\pi G_N)$, $k_2 = 1/(4\pi)$)

$$\begin{aligned} \mathcal{L}_t = & F_{\alpha\beta}^a F_{\gamma\delta}^b \eta^{\beta\delta} [\kappa_1 \xi_a^i \xi_b^j (\eta^{\alpha\gamma} \eta_{\varepsilon\theta} h_i^\varepsilon h_j^\theta + 2h_i^\gamma h_j^\alpha - 4h_i^\alpha h_j^\gamma) + \\ & \kappa_2 \eta_{cd} \eta^{\alpha\gamma} (\delta_a^c - \xi_a^i h_i^\varepsilon \beta_\varepsilon^c) (\delta_b^d - \xi_b^j h_j^\theta \beta_\theta^d)] / 4 + \mathcal{L}(\Psi, D_\alpha \Psi), \end{aligned} \quad (4.18)$$

where $\eta_{ab}(x)$ are the components of the symmetric undegenerate tensor fields. Now the Einstein equations, got from

$$\Phi_\alpha^j \left(\frac{\mathcal{L}_t}{\eta} \frac{\partial \eta}{\partial B_\alpha^b} + \frac{\partial \mathcal{L}_t}{\partial B_\alpha^b} - \nabla_i \left(\frac{\partial \mathcal{L}_t}{\partial \nabla_i B_\alpha^b} \right) \right) B_\beta^b \Phi_k^\beta = 0, \quad (4.19)$$

are written as

$$g^{ji} R_{ki} - \frac{1}{2} \delta_k^j R = \frac{1}{2\kappa_1} [D_a^{ij} E_{ik}^a - \frac{1}{4} \delta_k^j D_a^{il} E_{il}^a + P^j \Psi D_k \Psi - \delta_k^j \mathcal{L}(\Psi, D_i \Psi)], \quad (4.20)$$

where

$$D_i \Psi = \Phi_i^\alpha D_\alpha \Psi = \nabla_i \Psi - B_i^a L_a \Psi, \quad (4.21)$$

$$P^k \Psi = \frac{\partial \mathcal{L}}{\partial D_k \Psi} = \Phi_\alpha^k \frac{\partial \mathcal{L}}{\partial D_\alpha \Psi}, \quad (4.22)$$

$$B_i^a = \Phi_i^\alpha B_\alpha^a, \quad (4.23)$$

$$E_{ij}^a = (\delta_b^a - \xi_b^k B_k^a) (\nabla_i B_j^b - \nabla_j B_i^b + B_i^c B_j^d C_{cd}^b), \quad (4.24)$$

$$D_a^{ij} = \kappa_2 g^{ik} g^{jl} \eta_{cd} (\delta_a^c - \xi_a^p h_p^\varepsilon \beta_\varepsilon^c) (\delta_b^d - \xi_b^q h_q^\theta \beta_\theta^d) E_{kl}^b. \quad (4.25)$$

By this it can rewrite the total Lagrangian \mathcal{L}_t (4.18) as

$$\mathcal{L}_t = (H_k^{ij} F_{ij}^k + D_a^{ij} E_{ij}^a)/4 + \mathcal{L}(\Psi, D_\alpha \Psi), \quad (4.26)$$

where

$$F_{ij}^k = -\Phi_\alpha^k (\nabla_i \Phi_j^\alpha - \nabla_j \Phi_i^\alpha) \quad (4.27)$$

and

$$H_k^{ij} = \kappa_1 (g_{kl} F_{pq}^l g^{ip} g^{jq} + F_{kl}^i g^{jl} + g^{il} F_{ik}^j + 2g^{ip} \delta_k^j F_{lp}^l + 2\delta_k^i g^{jp} F_{pl}^l). \quad (4.28)$$

In the general case the condition (4.8) it is necessary to abolish ($L_{c\alpha}^\beta \neq 0$) so that the intensities $F_{\alpha\beta}^\nu$ of the fields $\Phi(x)$ will have the following form

$$F_{\alpha\beta}^\nu = \Phi_k^\nu (\Phi_\alpha^i \nabla_i \Phi_\beta^k - \Phi_\beta^i \nabla_i \Phi_\alpha^k) + B_\alpha^c L_{c\beta}^\nu - B_\beta^c L_{c\alpha}^\nu. \quad (4.29)$$

Already because of this the masses of the vector bosons (being the quanta of the gauge fields) can be the non-zero ones. Thus the interactions of the elemental particles with fields Φ_α^i which are describing the vacuum oscilations and which are connected with the gravitational interactions can lead to the appearance of the masses both of the fermions and of the vector bosons.

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